Optimization of Execution Time, Energy Consumption, and Accuracy during Finite Element Method Simulations

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Motivation

Finite Element Method (FEM) simulations

significant percent of simulations in supercomputing centres

- ABAQUS

„widely used in automotive, metallurgical, mining, shipbuilding and aerospace engineering industry”

- ANSYS

„comprehensive simulations in almost every field of science and industry”

- COMSOL Multiphysics

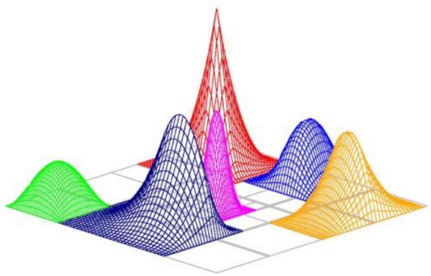
„used in different areas of engineering and scientific applications (electronics, mechanics, chemical enginering, geophysics …)”

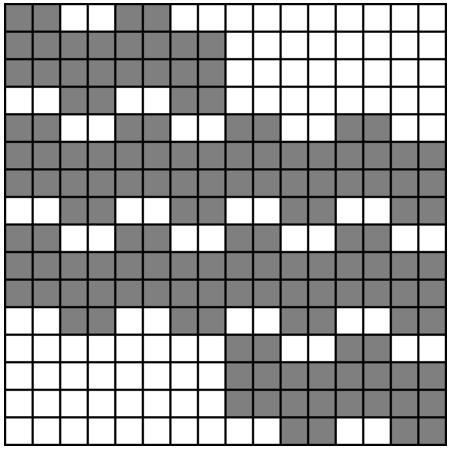
- OPERA

„Computational package using FEM for prerforming simulations of electromagnetic waves propagation in complex physical objects”

The goal of our research is to optimize the finite element method simulations for energy consumption, execution time, and accuracy of the numerical solution

Finite element method: mesh, basis functions, system of linear equations



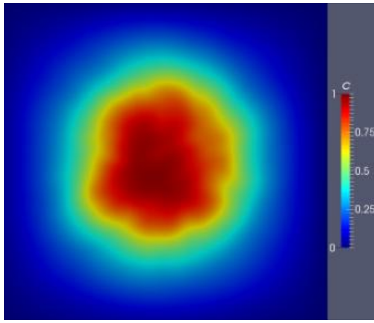


Direct solvers
MUMPS, PASTIX
SuperLU, etc.

Iterative solvers
PCG, GMRES,
ILUPCG, etc.

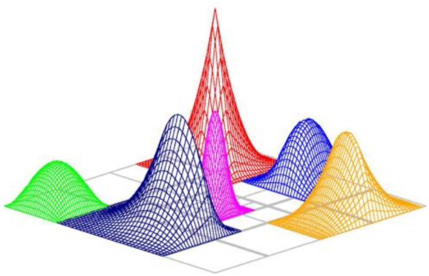
Alternating
directions solvers
(time-dependent)

H-matrix solvers



Simulated phenomenon is approximated by basis functions spread over the computational mesh

Finite element method: mesh, basis functions, system of linear equations



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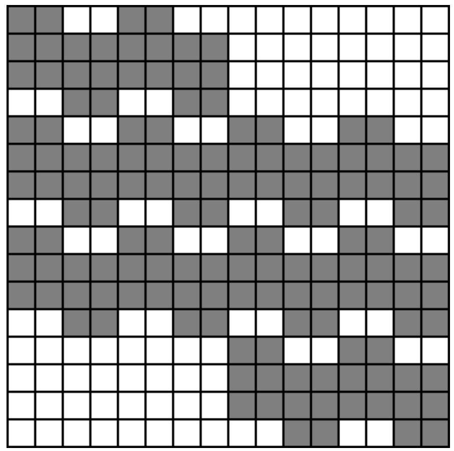

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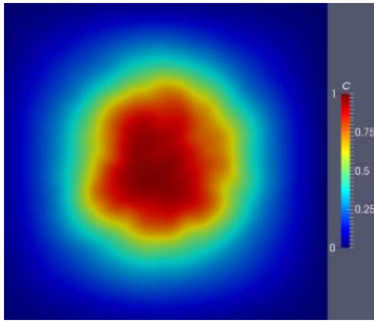
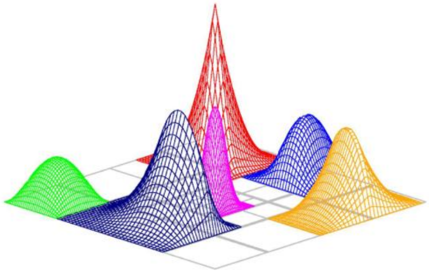


System of linear equations =rows and columns represent basis functions
Non-zero values ← interactions of basis functions
(supports of basis functions overlap on computational mesh)

Finite element method: mesh, basis functions, system of linear equations



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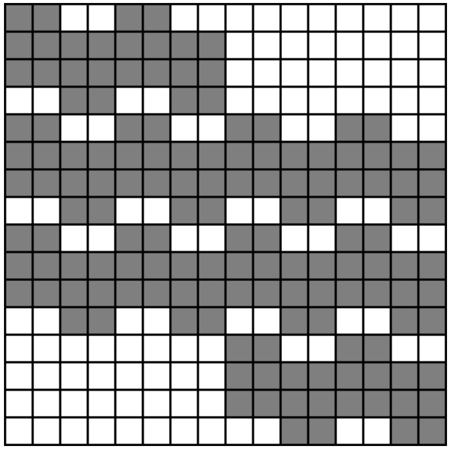

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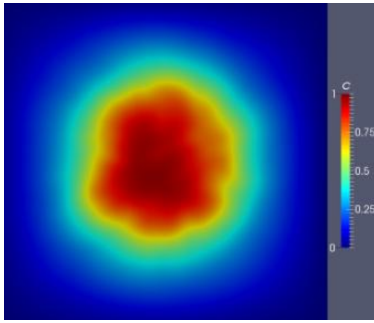
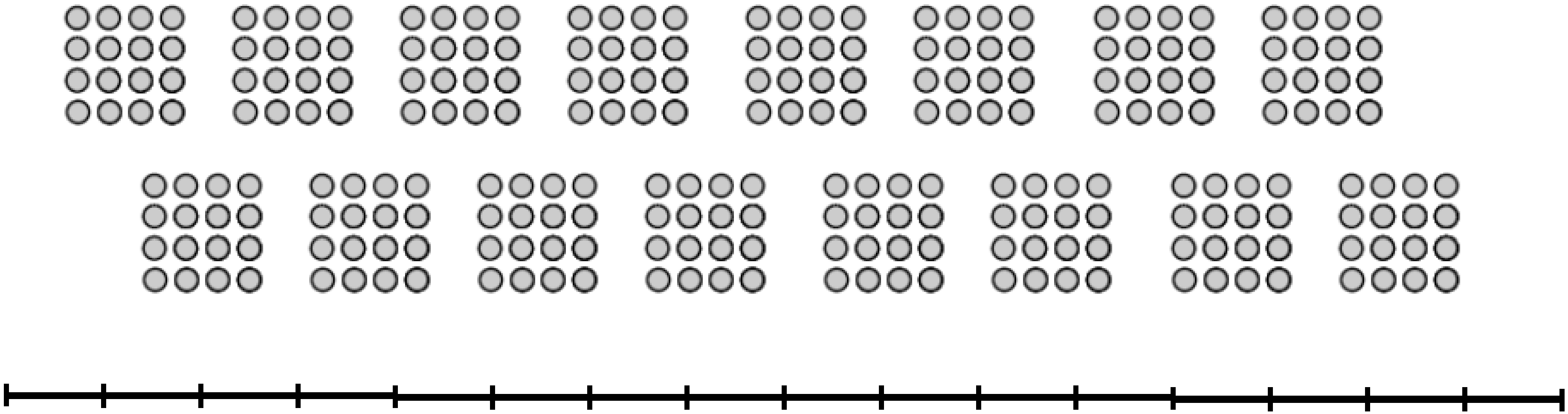


Computational cost of finite element method
= cost of solution of the system of linear equations

Optimization of direct solvers

Exemplary optimalization of direct solver



16 finite elements, 16 element matrices

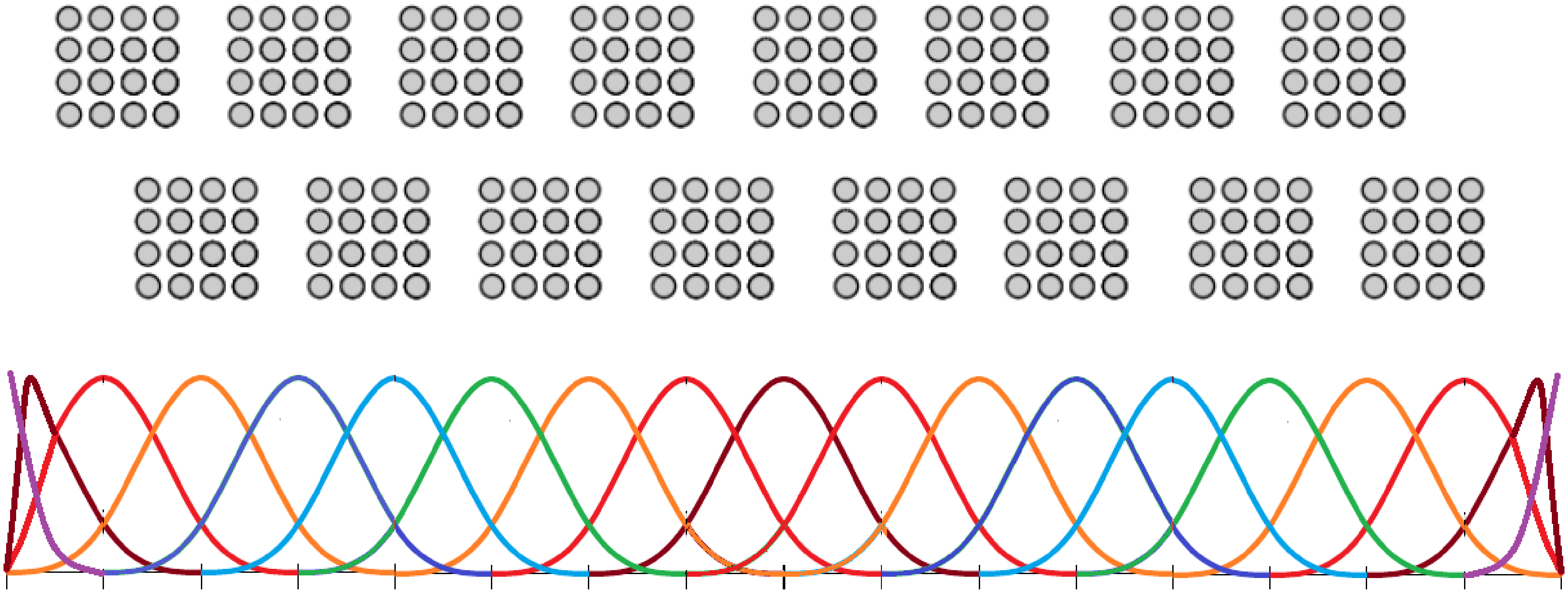
assembled into

1 global matrix

factorized by

Direct solver

Direct solver optimization
B-spline basis functions



16 elements with cubic B-splines

4 basis functions per element → element matrices 4x4

Direct solver optimization
B-spline basis functions

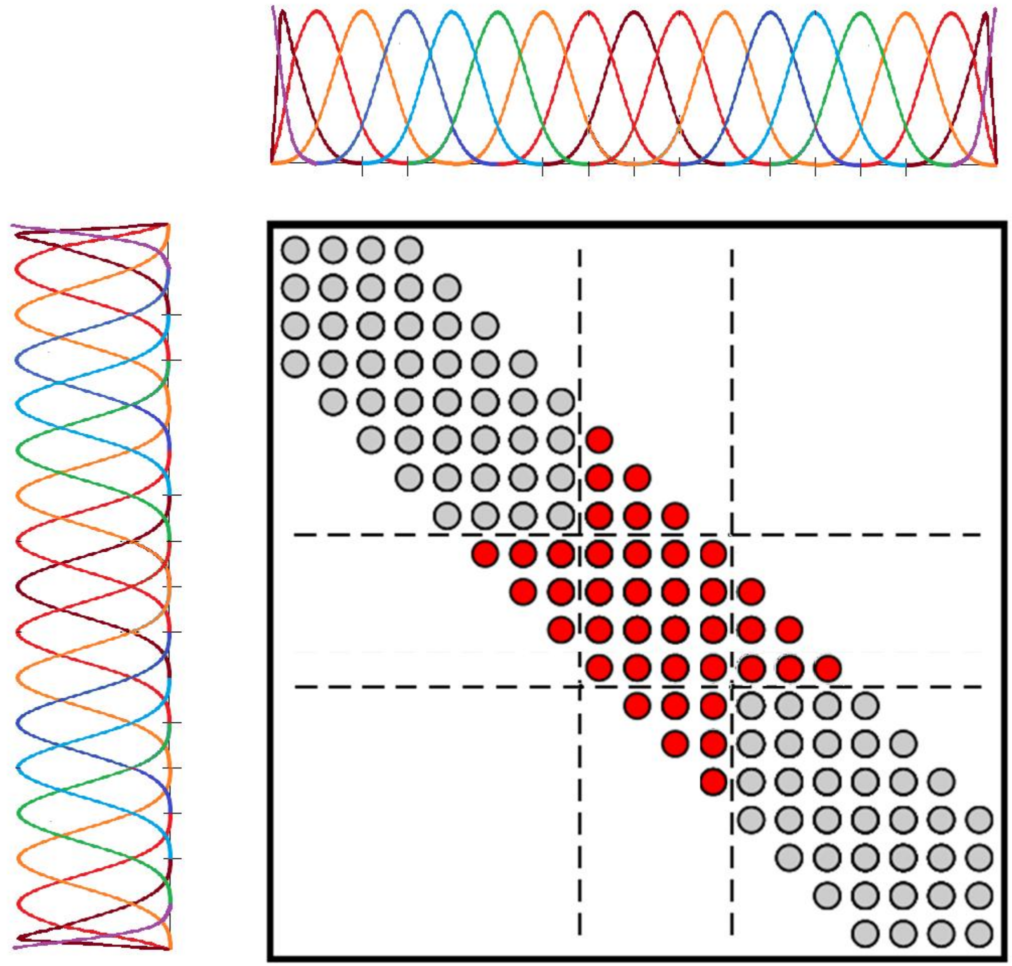
16 element matrices 4x4

↓ assembled into

Global matrix

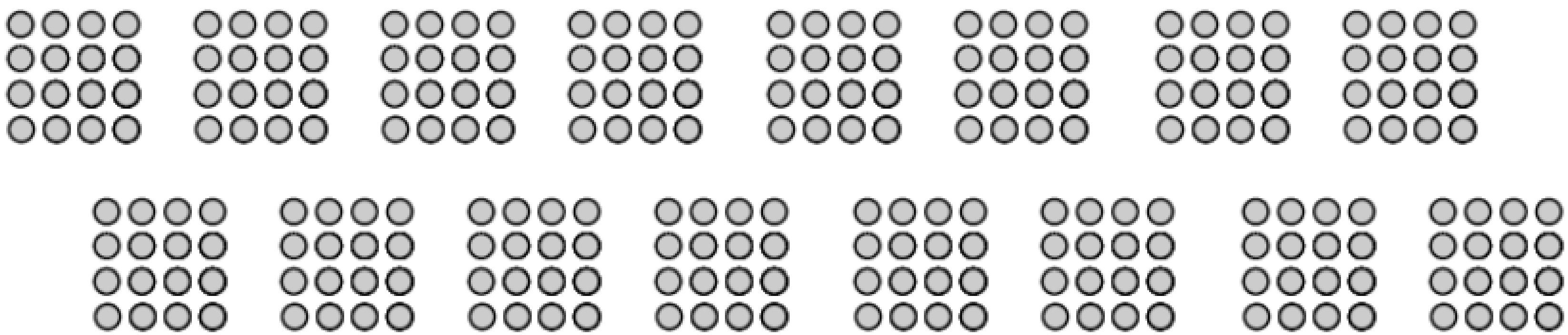
Small matrix size N=19
(=16+3)

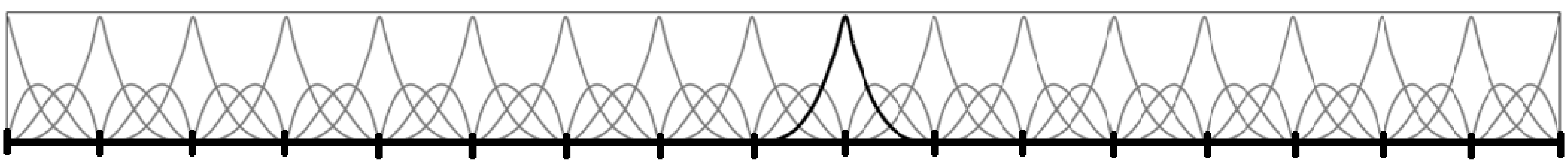
Dense on diagonals



Element matrices overlap to the greatest extend

Direct solver optimization: Lagrange'a polynomials





Lagrange'a polynomials used in traditional FEM can be obtained by introduction of C^0 separators between elements

Direct solver optimization: Lagrange'a polynomials

16 element matrices 4x4

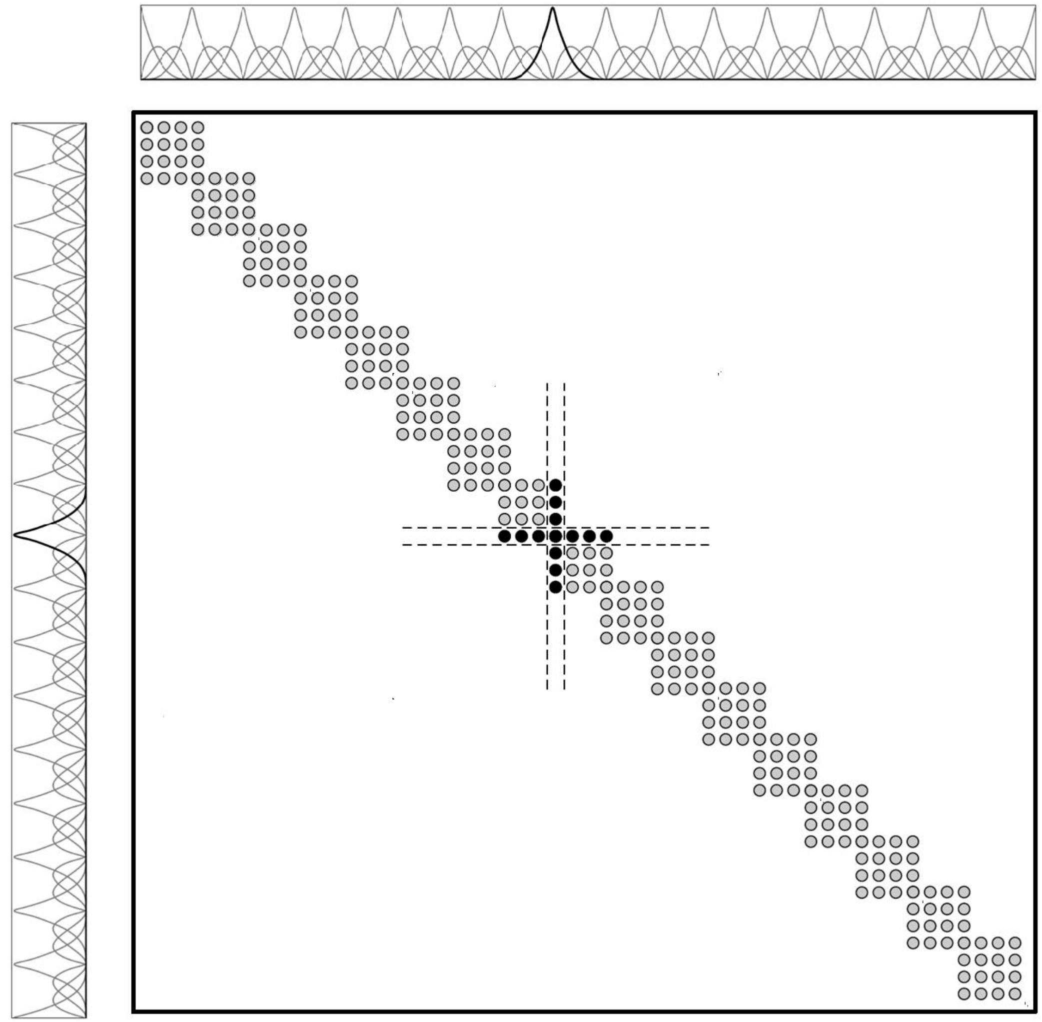
↓ assembled into

Global matrix

Large size N=49

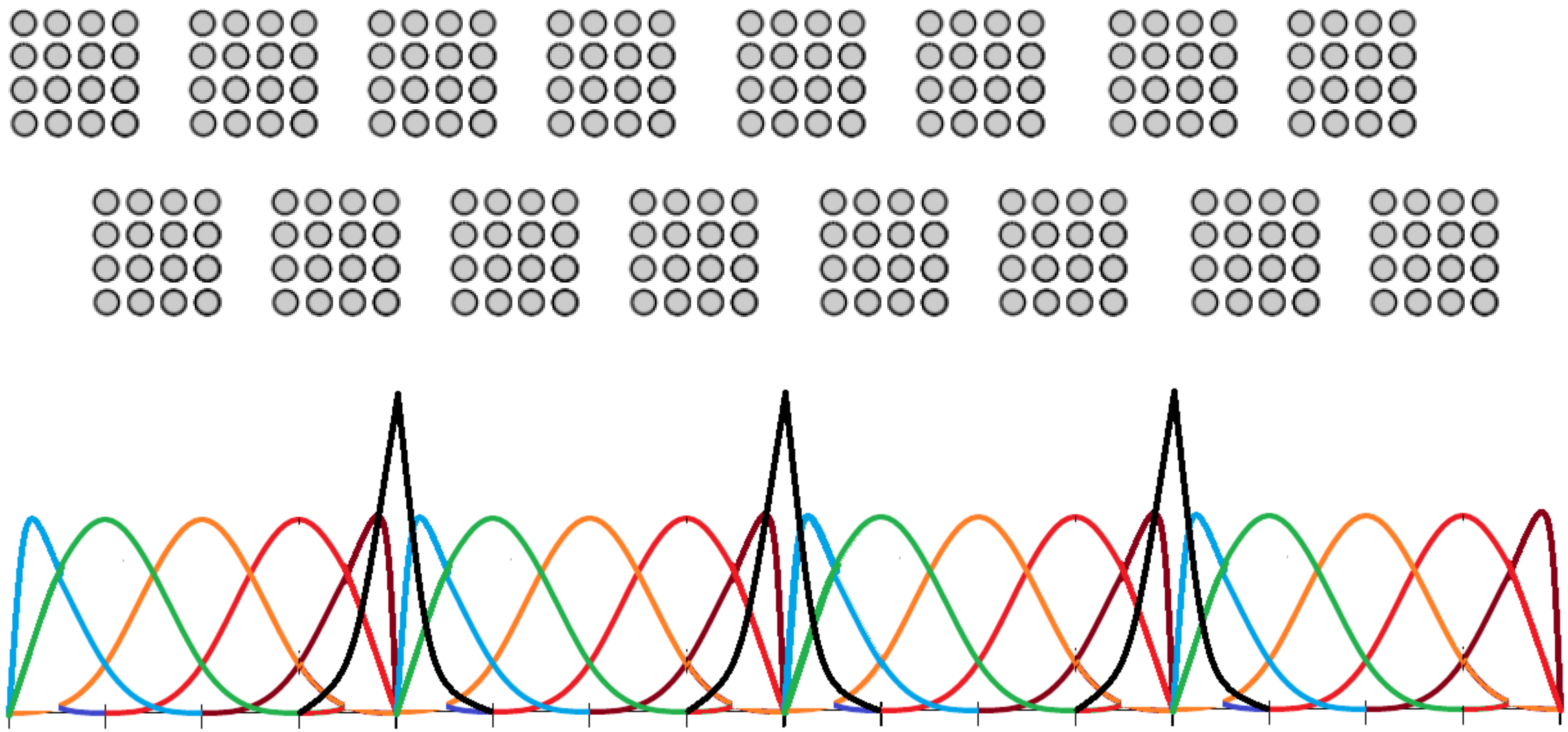
(=3*16+1)

Sparse on diagonals



Element matrices overlap in minimal way

Direct solver optimization: refined Isogeometric Analysis (**rIGA**)



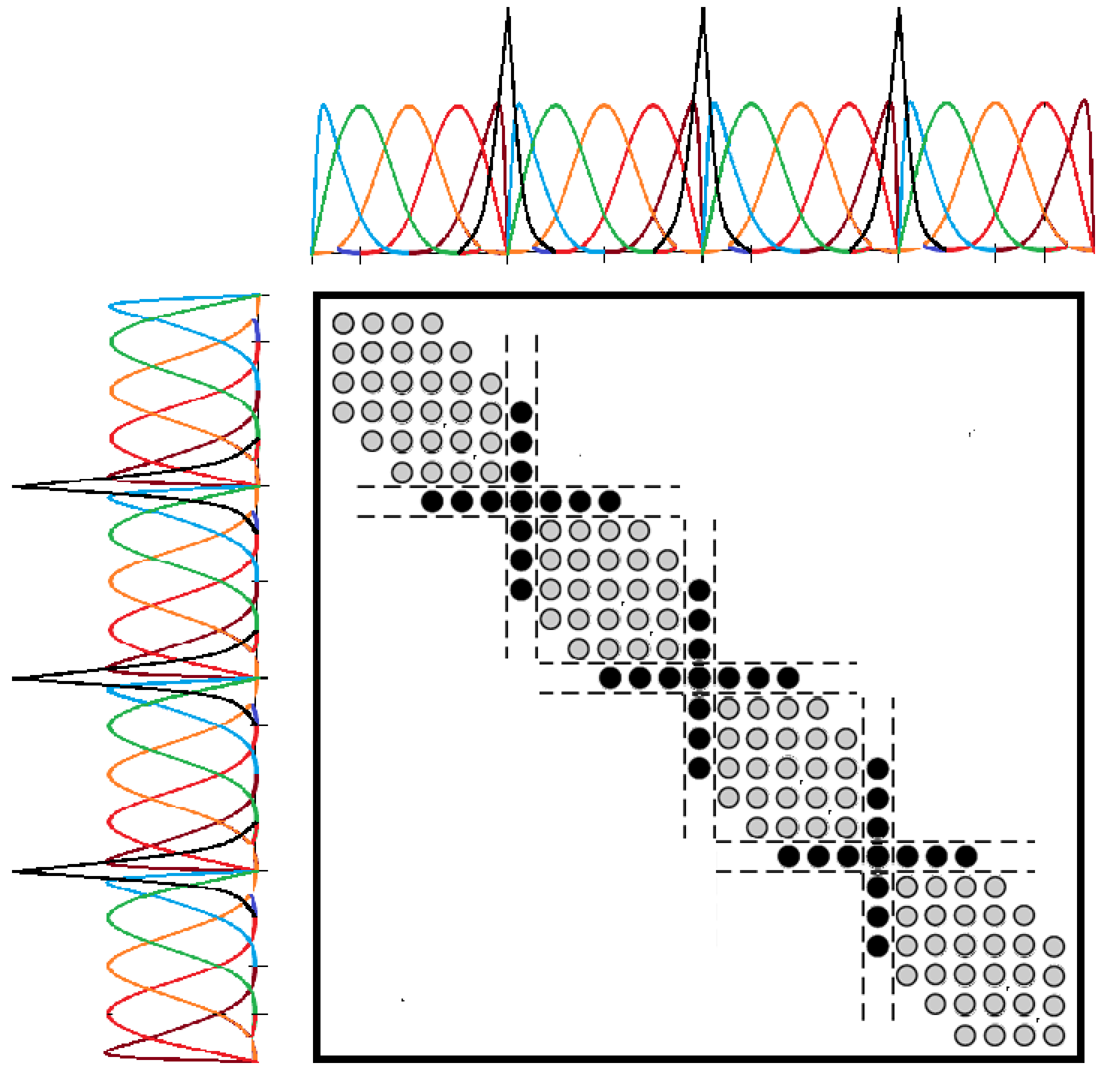
Compromise between both methods
16 elements with cubic B-splines
with additional C0 separators every 4 elements

Direct solver optimization: refined Isogeometric Analysis (**rIGA**)

16 element matrices 4x4

assembled into

Global matrix

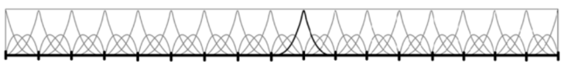


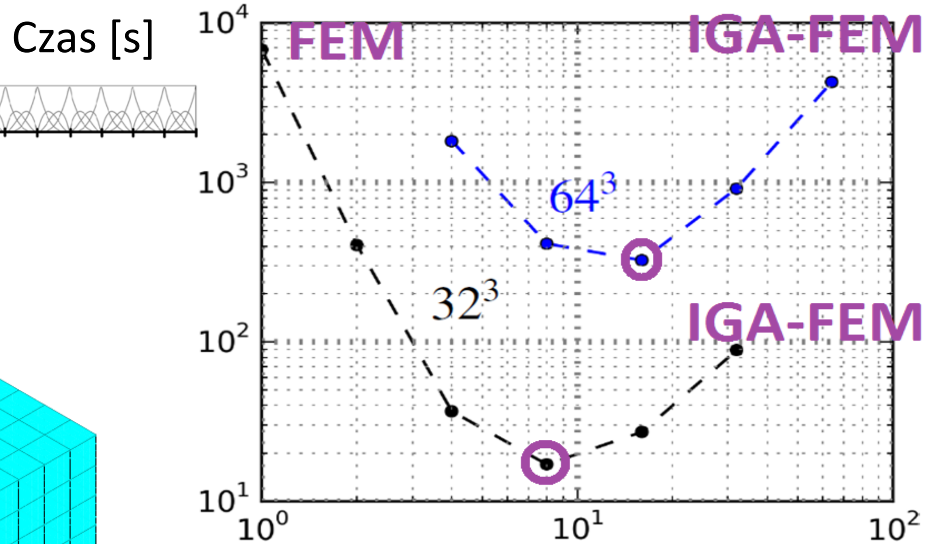
Average size N=25

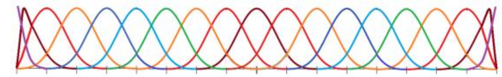
(=4*(4+2)+1)

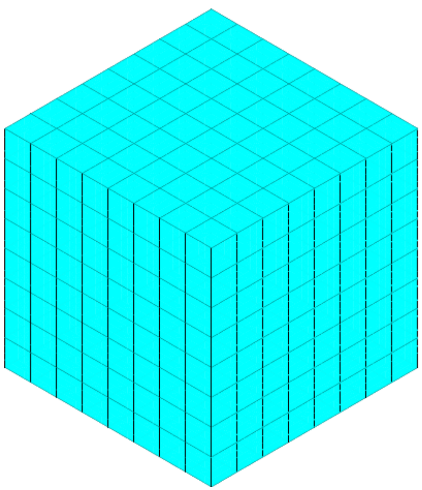
Average density

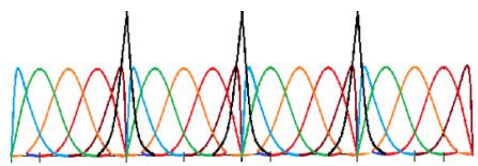
Optimization of basis functions for direct solvers









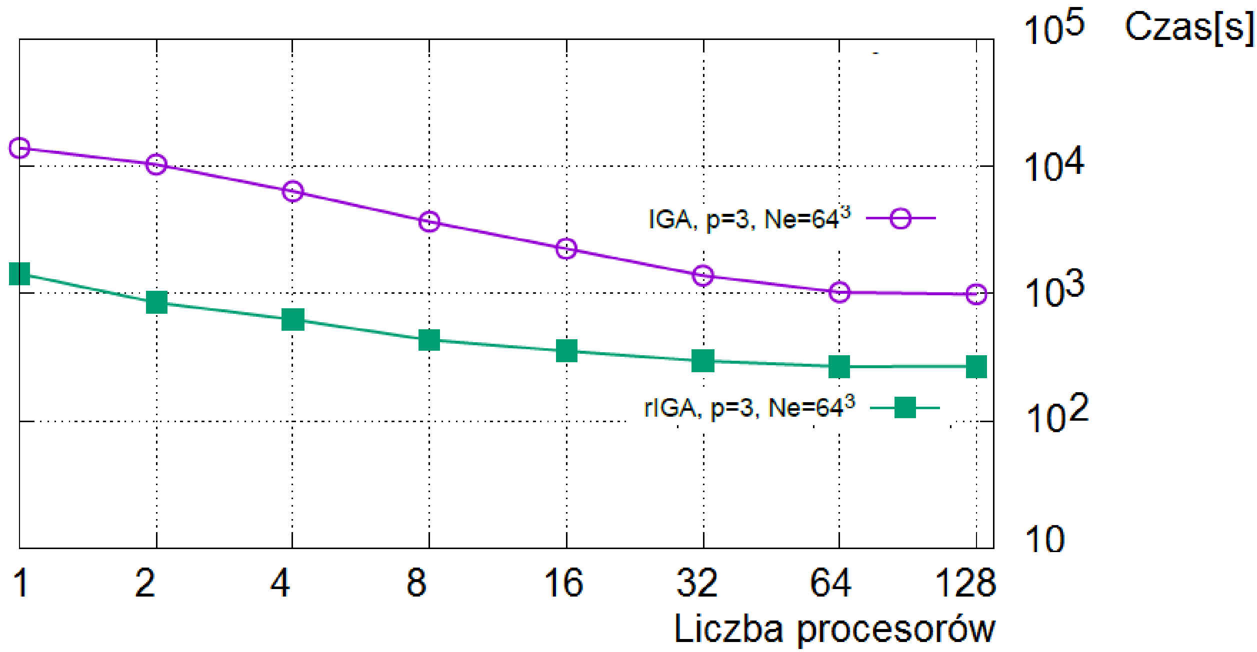


29 speedup with respect to Lagrange'a basis

12 speedup with respect to B-spline basis

Daniel Garcia, David Pardo, Lisandro Dalcin, Maciej Paszynski, Victor M. Calo, Refined Isogeometric Analysis (rIGA): Fast Direct Solvers by Controlling Continuity, **Computer Methods in Applied Mechanics and Engineering** 316 (2017) 586-605

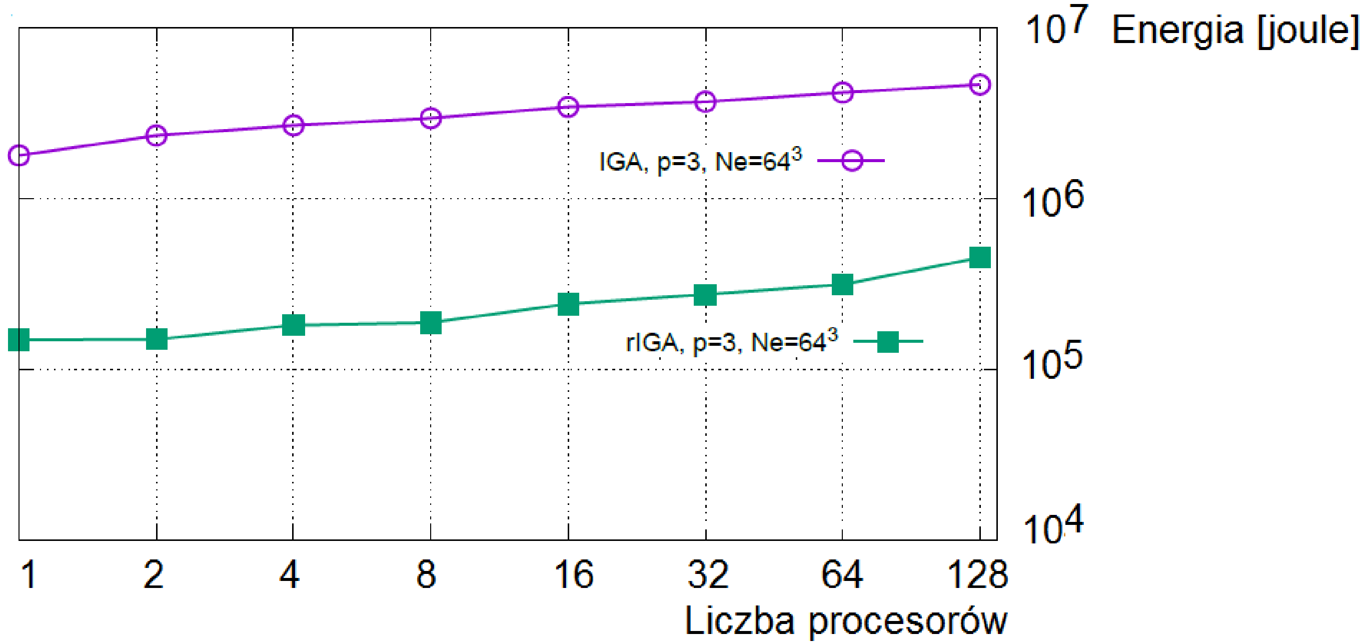
Optimization of execution time of parallel direct solvers



Application of basis functions from refined isogeometric analysis allows reduction of execution time **by one order of magnitude** using state-of-the-art parallel MUMPS solver

Maciej Paszyński, Maciej Woźniak, Leszek Siwik, Dariusz Król,
A simple trick to speed up your parallel isogeometric analysis simulations,
Parallel Computing (2017 submitted)

Optimization of energy consumption of parallel direct solvers



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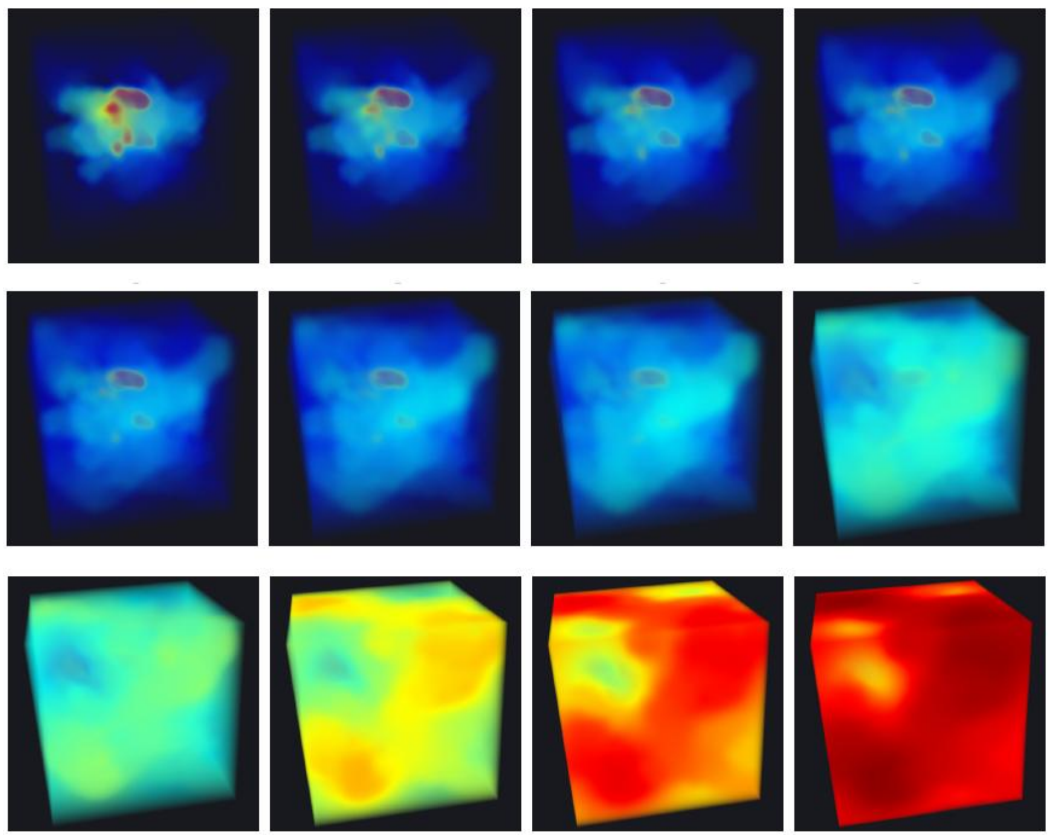
Optimization of alternating directions solvers

Exemplary simulations performed by using alternating directions solver

- **Simulations of extraction of oil formations by using hydraulic fracking**

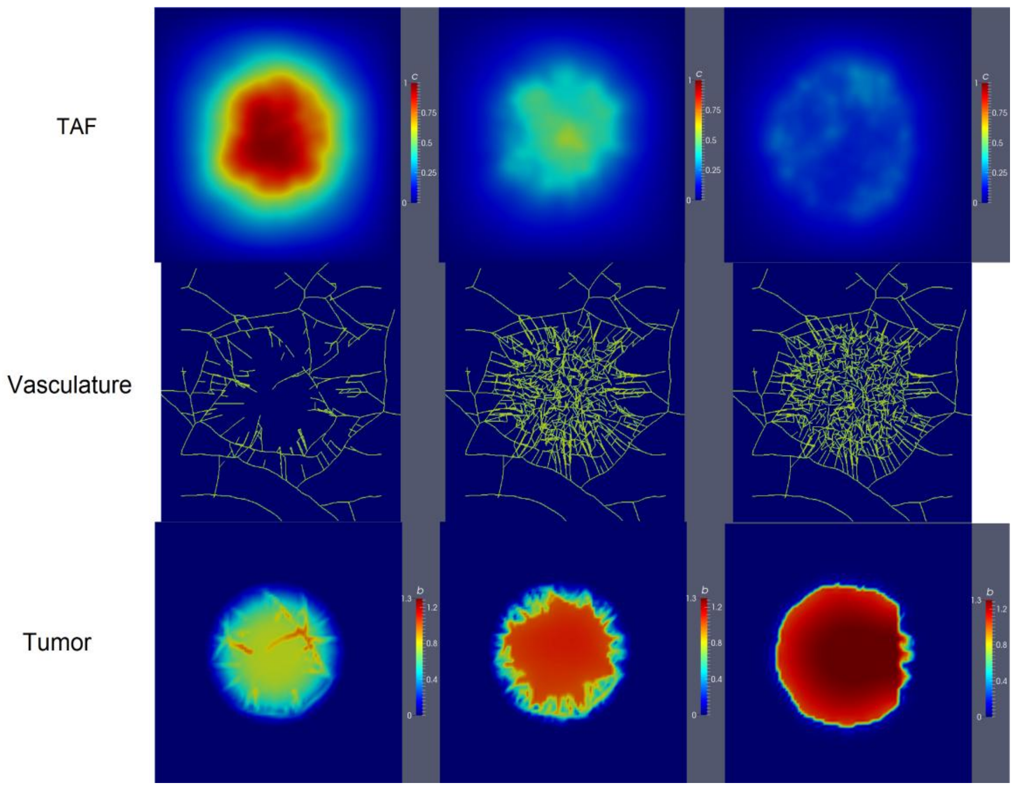
Marcin Łoś, Maciej Woźniak, Maciej Paszyński, Andrew Lenharth, Keshav Pingali,
IGA-ADS : Isogeometric Analysis FEM using ADS solver,
Computer & Physics Communications (2017) IF: 3.268

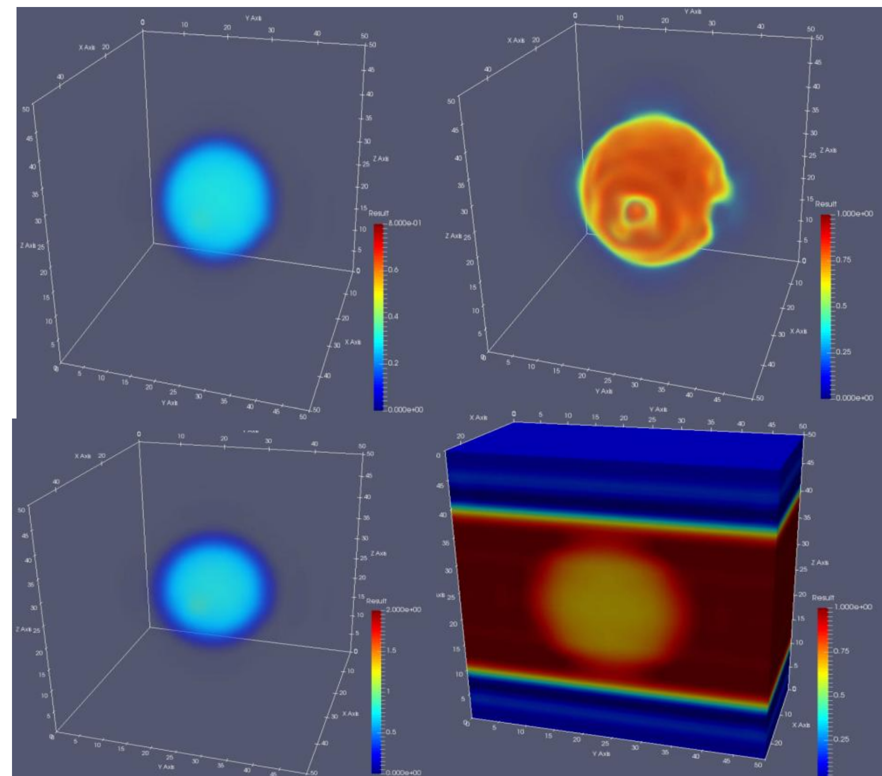


Exemplary simulations performed by using alternating directions solvers

- **Tumor growth simulations**

Marcin Łoś, Maciej Paszynski, Adrian Kłusek, Witold Dzwinel, Application of fast isogeometric L2 projection solver for tumor simulations,
Computer Methods in Applied Mechanics and Engineering (2017) IF: 3.915





Optimization of alternating directions solver

Time dependent problems, explicite method

Two fold increase of the mesh size in every dimension implies

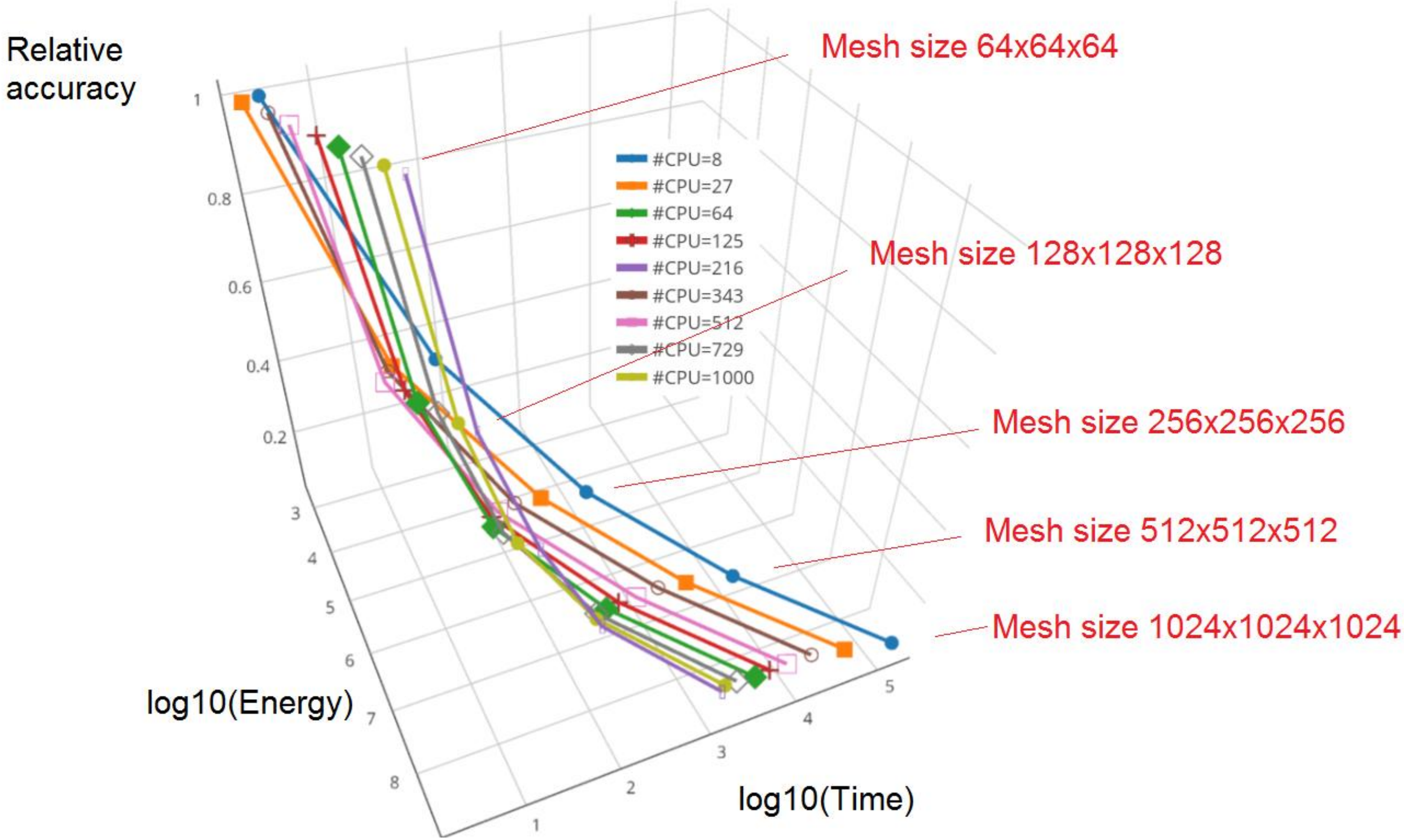
→ reduction of the numerical error according to

$$\|u - u_h\|_E \leq Ch^{p+1-m}\|u\|_{p+1}$$

here *h* stands for mesh diameter,
p stands for B-splines order
m stands for the PDE order

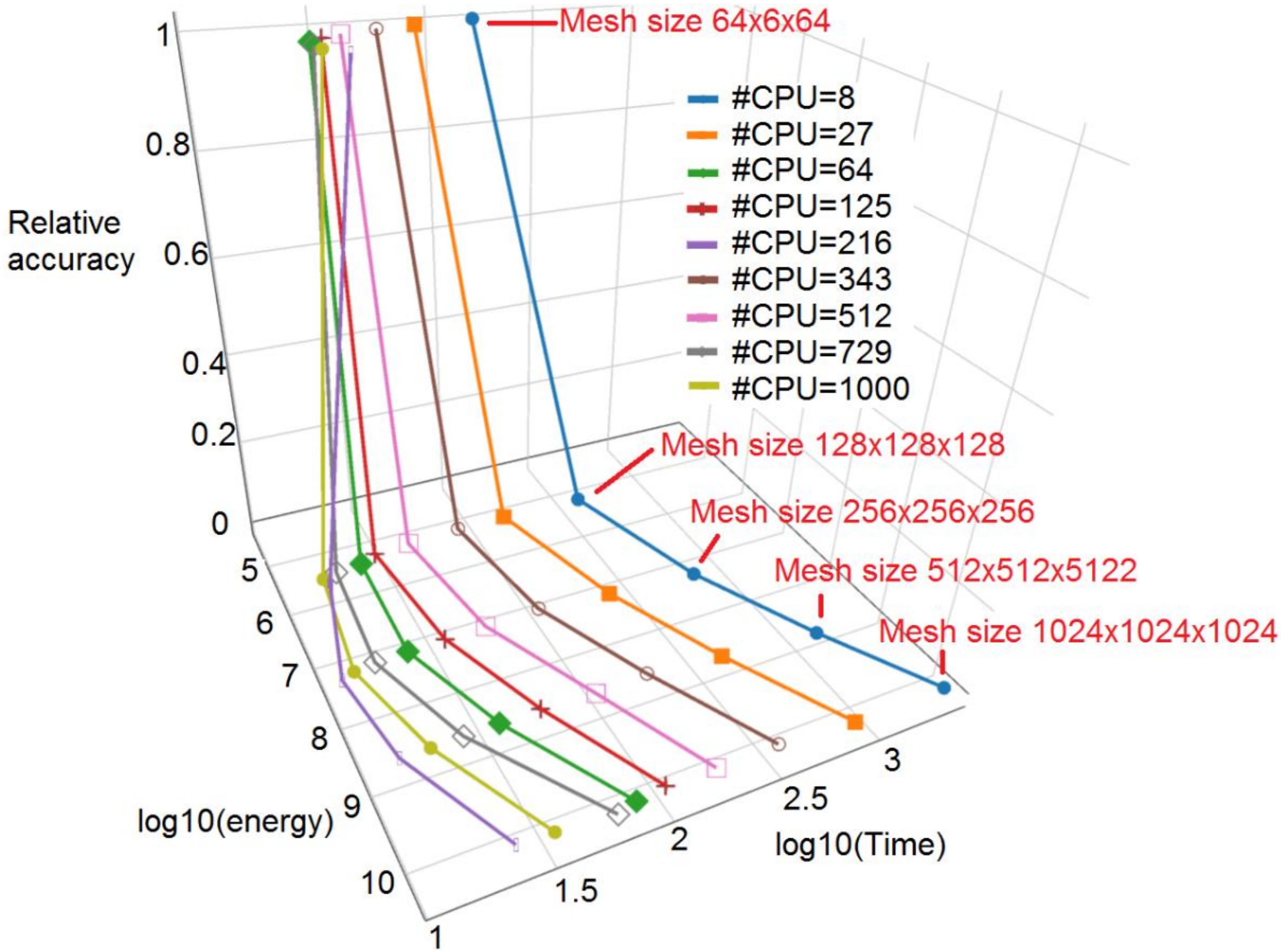
→ quadruple increase of the number of time steps according to Courant-Friedrichs-Lewy (CFL) condition
U_x dt/dx + U_y dt/dy + U_z dt/dz < const

Optimization of alternating direction solvers



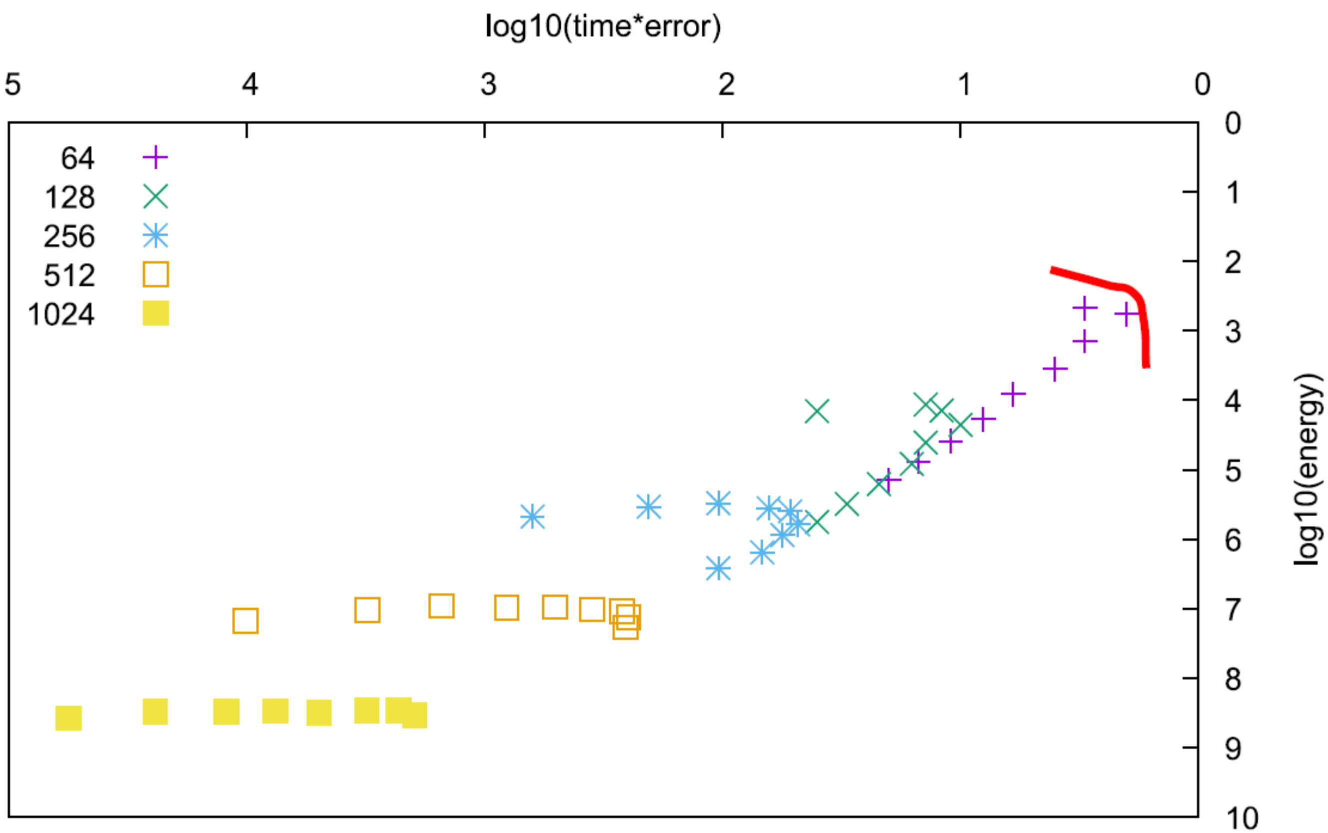
Quadratic B-splines

Optimization of alternating direction solvers



Quintic B-splines

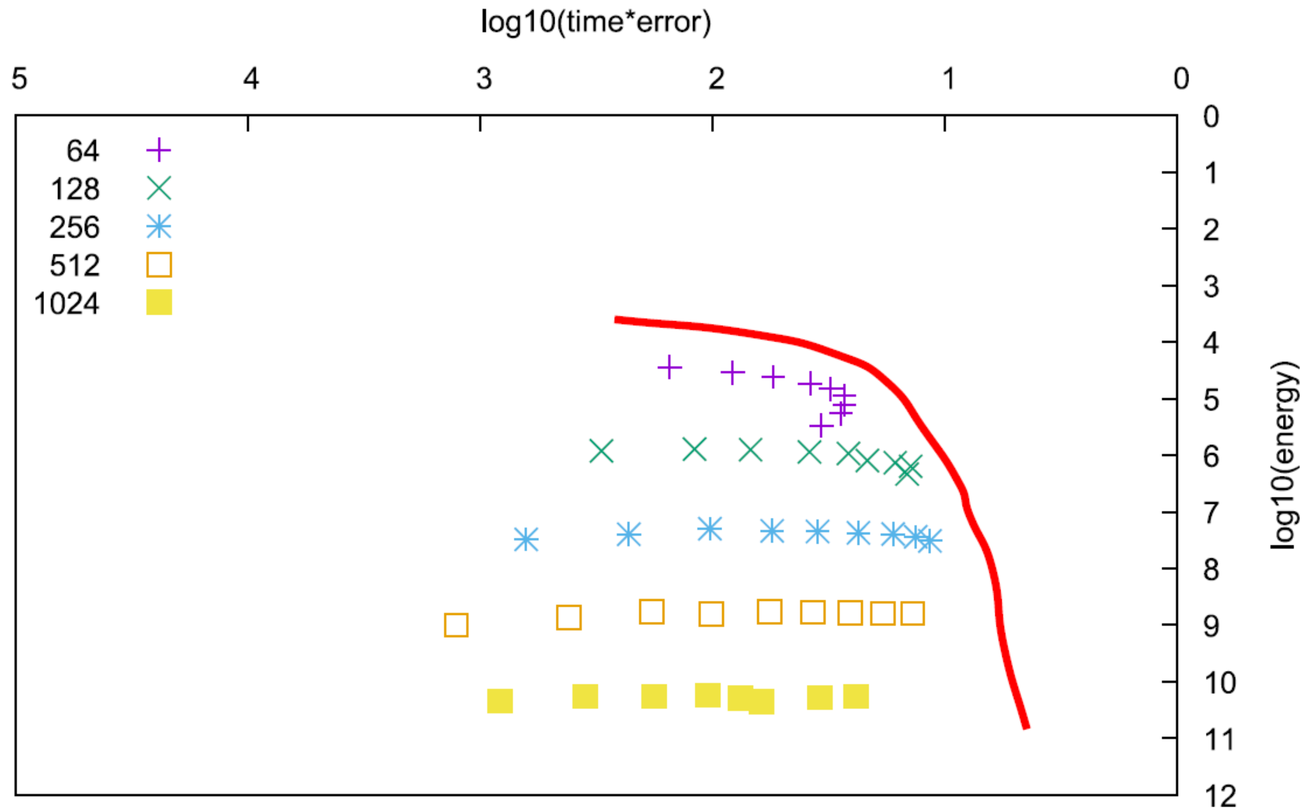
Optimization of alternating direction solvers



Pareto front for optimization of mesh size and number of processors with respect to energy consumption, execution time and accuracy

quadratic B-splines

Optimization of alternating direction solvers



Pareto front for optimization of mesh size and number of processors with respect to energy consumption, execution time and accuracy

quintic B-splines

Conclusions

- Smart modification of basis functions *reduces execution time and energy consumption of direct solves by one order of magnitude*
- Several choices of mesh size, polynomial order and number of processors *make no sense*, since they are dominated by optimal Pareto front choices

Future work:

- Construction of the expert system advertizing mesh-size, basis functions and number of procossors for requested accuracy to optimize energy consumption and execution time
- Optimization of iterative and H-matrix solvers

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