The new algorithm of description of multifractal cross-correlation

Paweł Oświęcimka

Stanisław Drożdż

Marcin Forczek

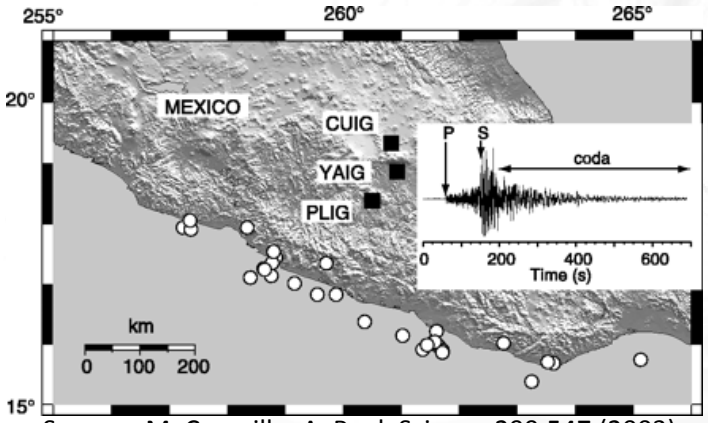
Instytut Fizyki jądrowej PAN

Politechnika Krakowska

Example of signals exhibit cross-correlation

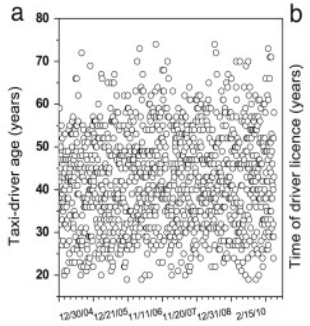
Geophysics

Seismic waves

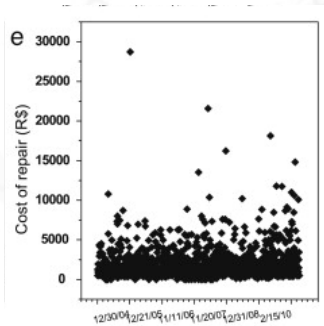


Source: M. Campillo, A. Paul, Science 299,547 (2003)

Traffic flows



Taxi driver age

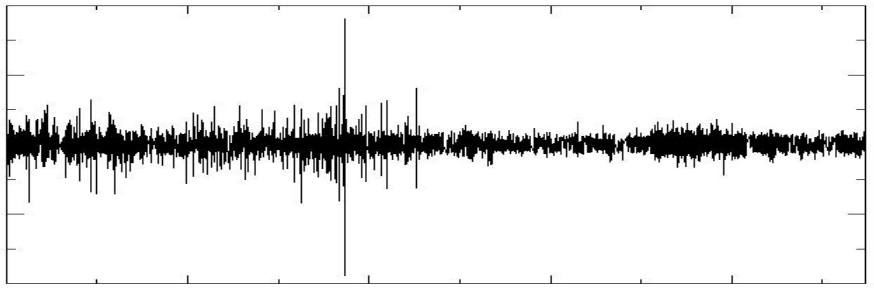


Cost of taxi repair

Source: G.F. Zebende, et al. Physica A 390, 1677-1683 (2011)

Finance

Price fluctuations



Correlation matrix ***C***

$$C_{ij} = \frac{1}{T}\sum_{t=1}^{T} \delta x_i(t)\delta x_j(t)$$

Fractals and multifractals

Power Law – self-similarity

Monofractals

$$F \sim s^h$$

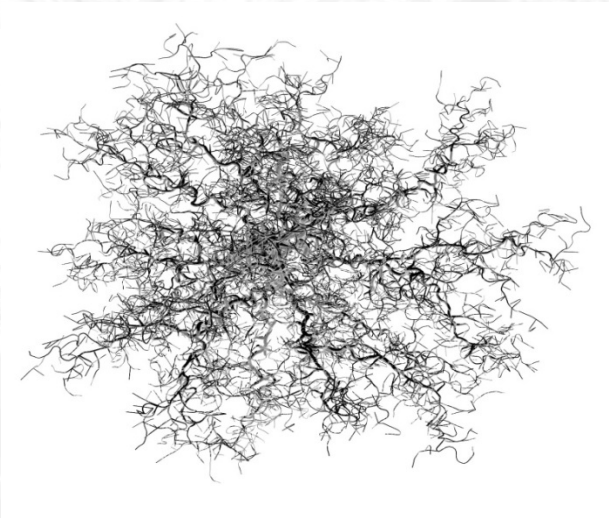
Multifractal patterns scale with multiple scaling rules rather than one global scaling rule

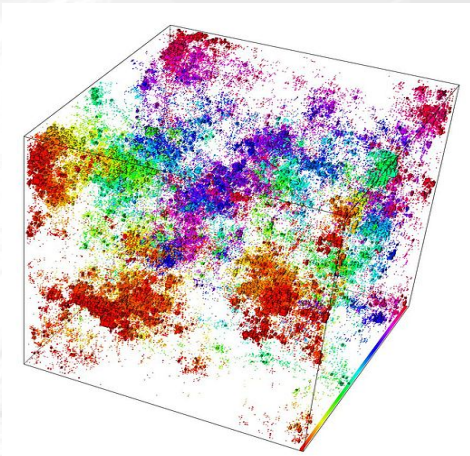
Multifractals

$$F(q,s) \sim s^{h(q)}$$

h(q)- generalized Hurst exponent

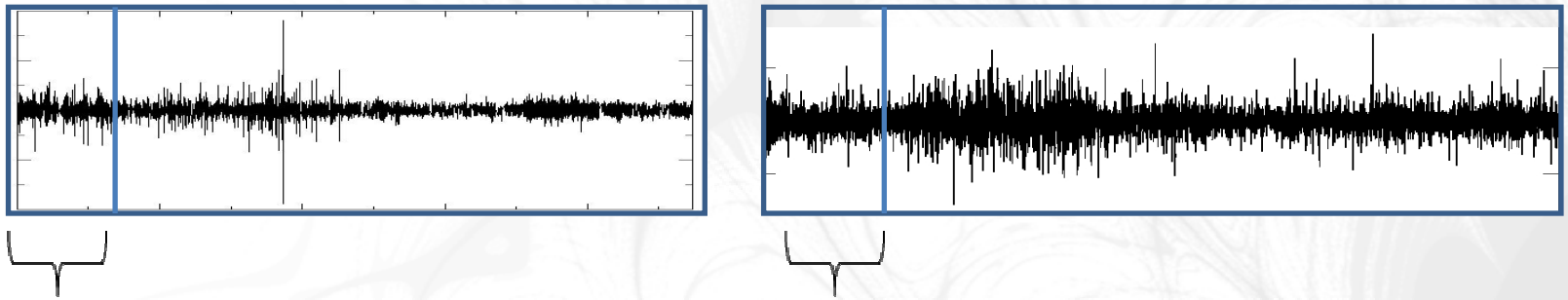
Diffusion limited aggregation





Source: http://en.wikipedia.org/wiki/Multifractal_system

Multifractal cross-correlation



$$F_v(s) = \frac{1}{s}\sum_{k=1}^{s}[X_v(s) - \tilde{X}][Y_v(s) - \tilde{Y}]$$

$$F_q(s) = \left(\frac{1}{m}\sum_{v=1}^{m} sign(F_v(s))|F_v(s)^{q/2}|\right)^{1/q}$$

$$F_q(s) \sim s^{\lambda(q)}$$

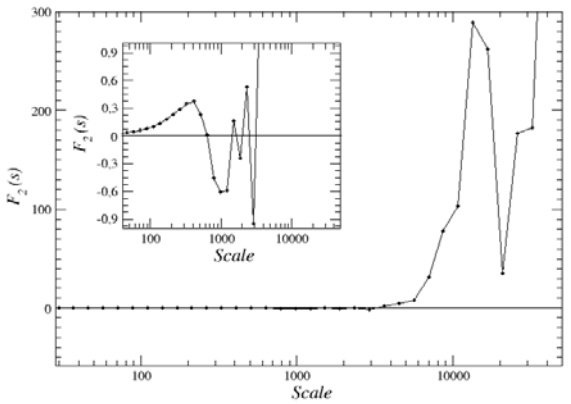
Detrended covariance

Decomposition of the covariance

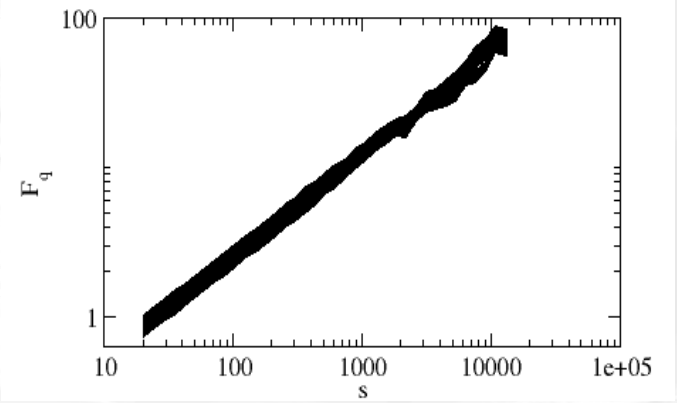
Determine the scaling behaviour

$$F_q(s) \sim s^{\lambda(q)}$$

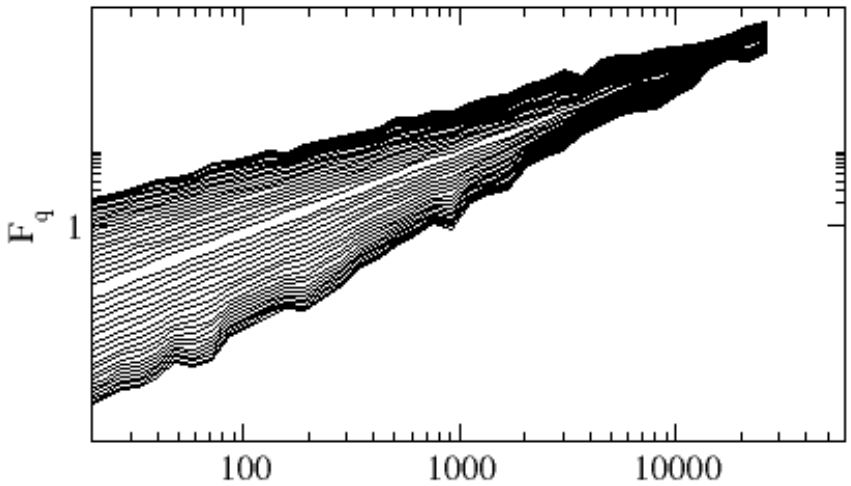
No fractal Cross-correlation



Monofractal Cross-correlation



Multifractal Cross-correlation



ARFIMA processes

$$y_i \equiv \sum_{j=1}^{\infty} a_j(\rho) y_{i-j} + \eta_i$$

$i.i.d$ *Gaussian variable*

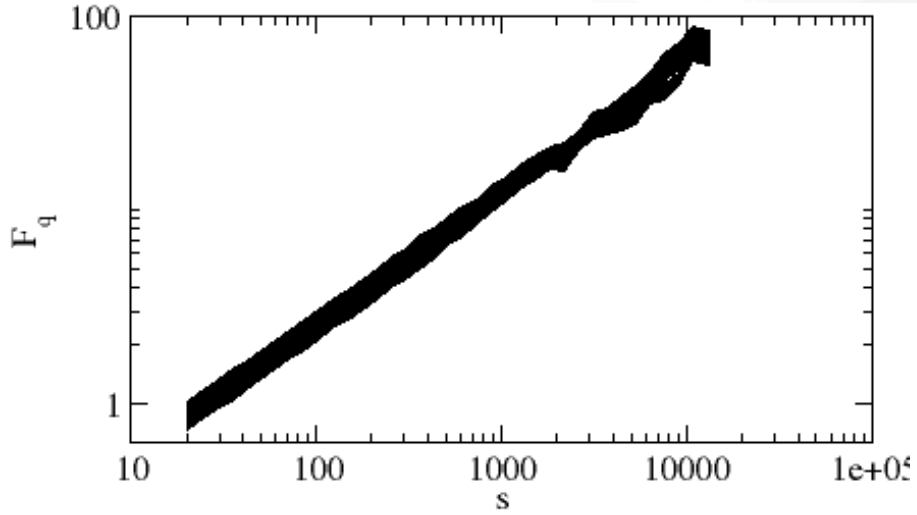
$$a_j(\rho) \equiv \Gamma(j-\rho)/[\Gamma(-\rho)\Gamma(1+j)]$$

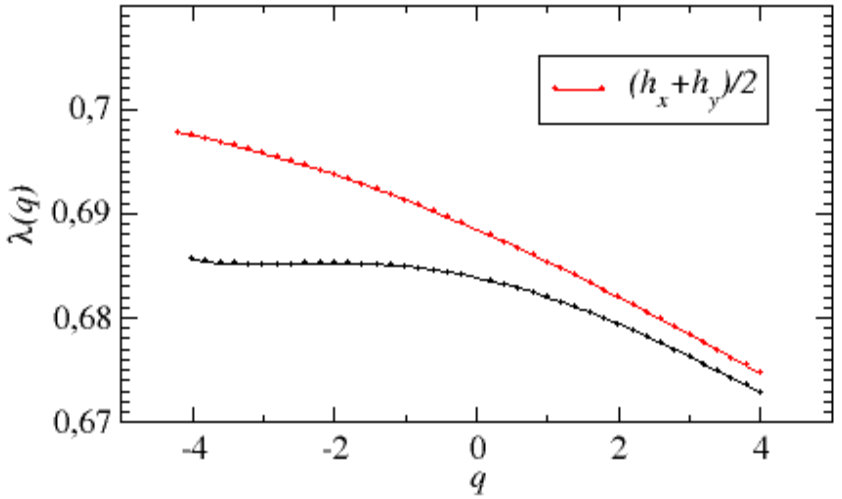
$$H = 0.5 + \rho$$

Hurst exponent

X time series *H=0.5*

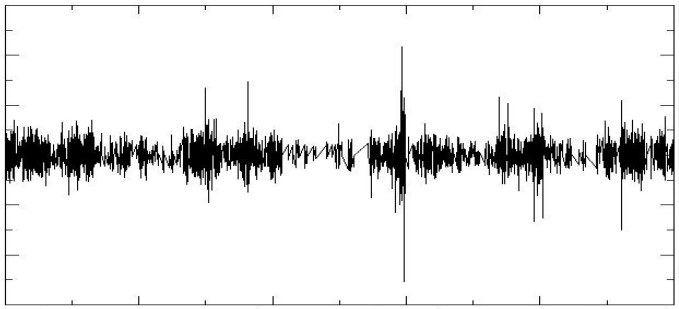
Y time series *H=0.9*





Monofractal cross-correlation characteristics

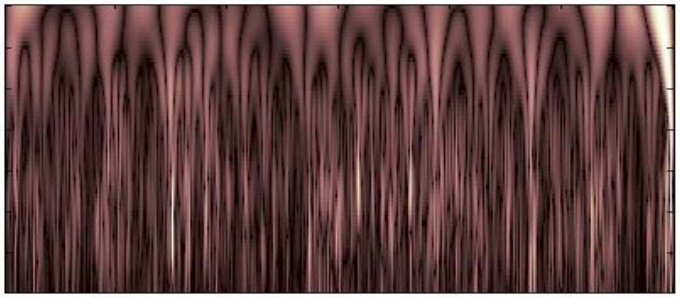
Markov-switching multifractal model



$$r_t = \sigma_t u_t$$

$$\sigma_t^2 = \sigma^2 \prod_{i=1}^{k} M_t^{(i)}$$

$i.i.d\ Gaussian\ variable$

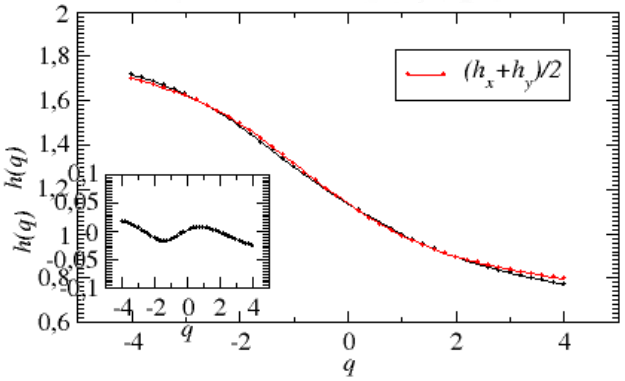


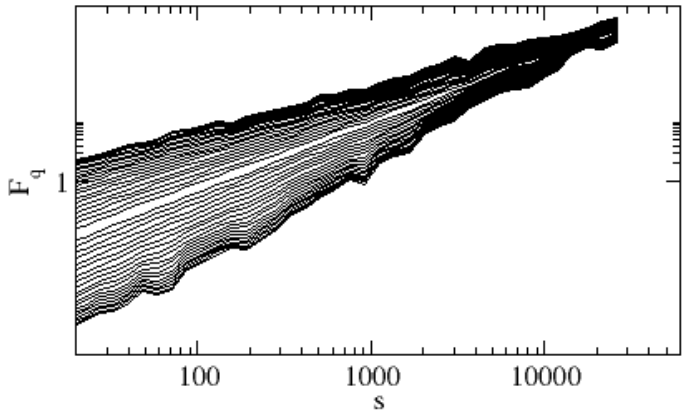
Scale

$$M_t^{(i)} \sim \{m_0, 2 - m_0\}$$

Time

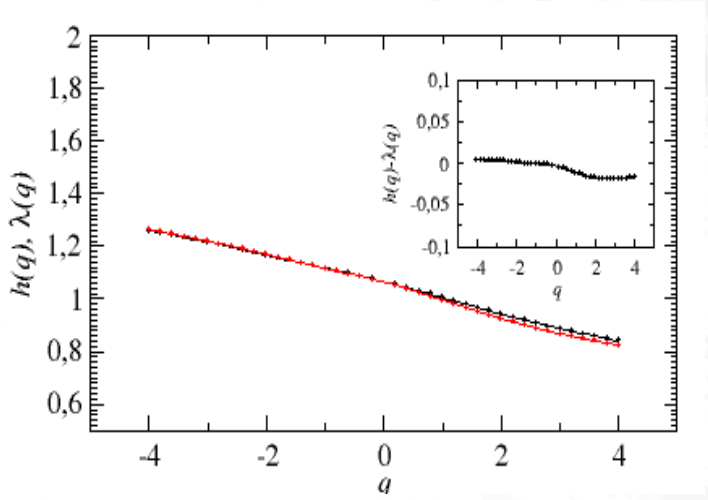
Multifractal cross-correlation behaviour



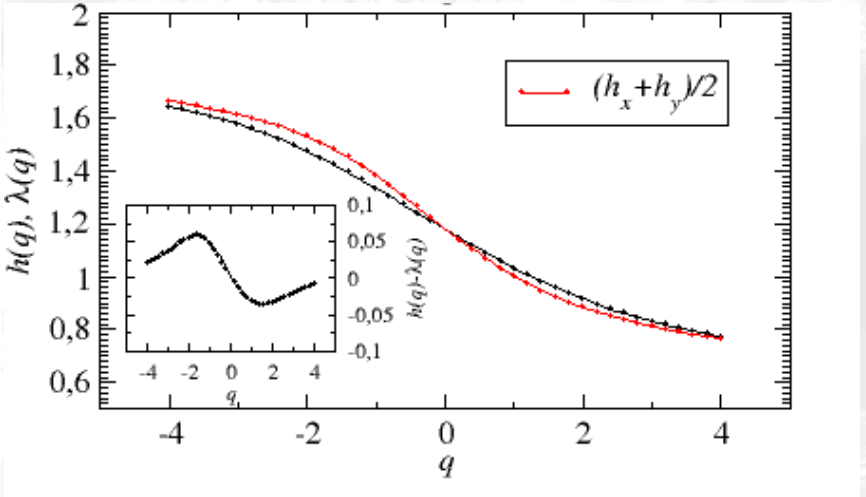


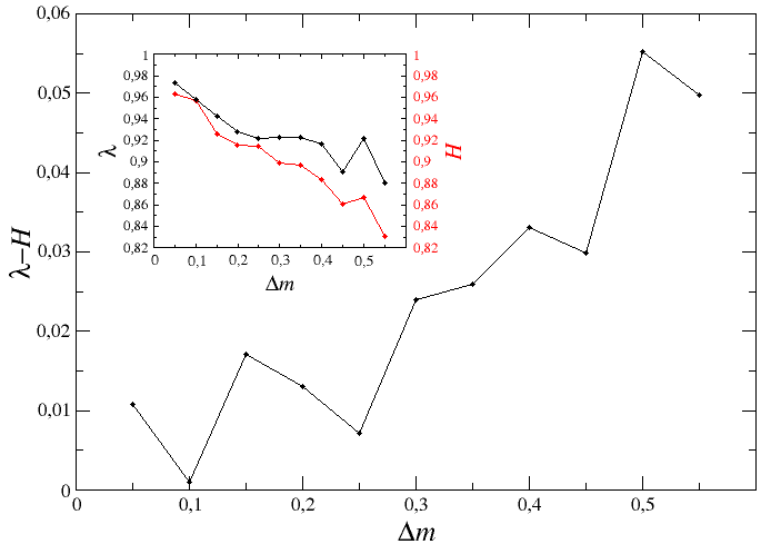
Markov-switching multifractal model

$\Delta m=0.1$



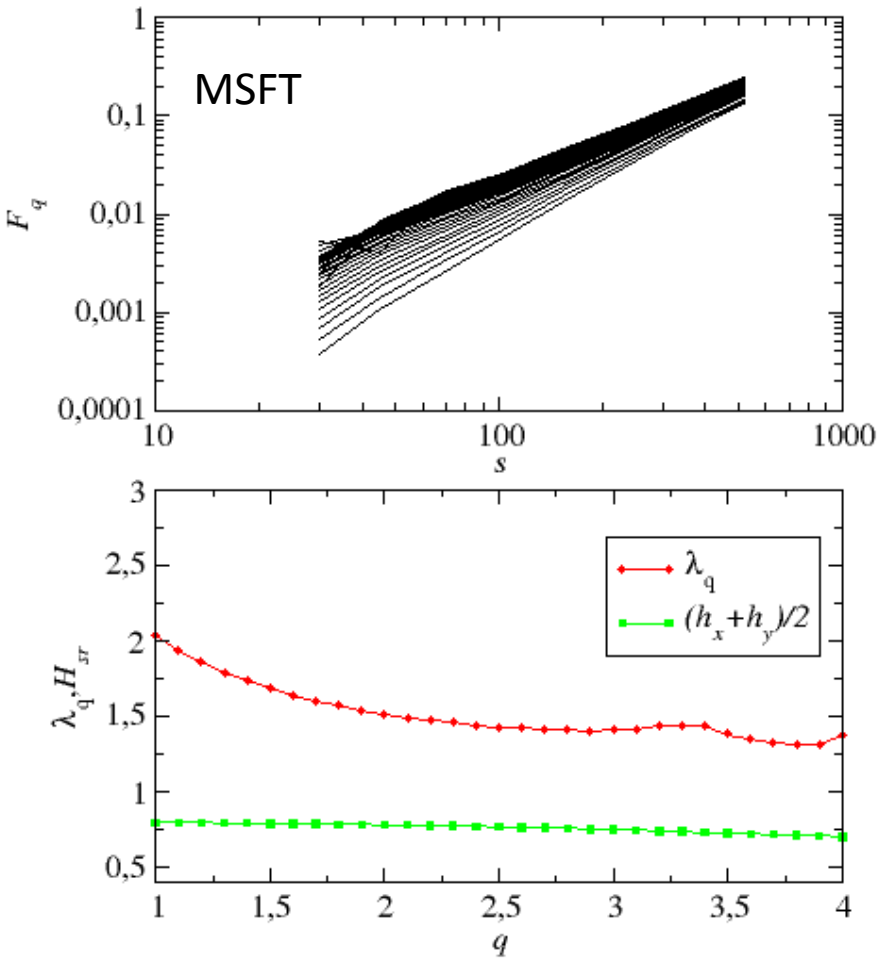
$\Delta m=0.5$



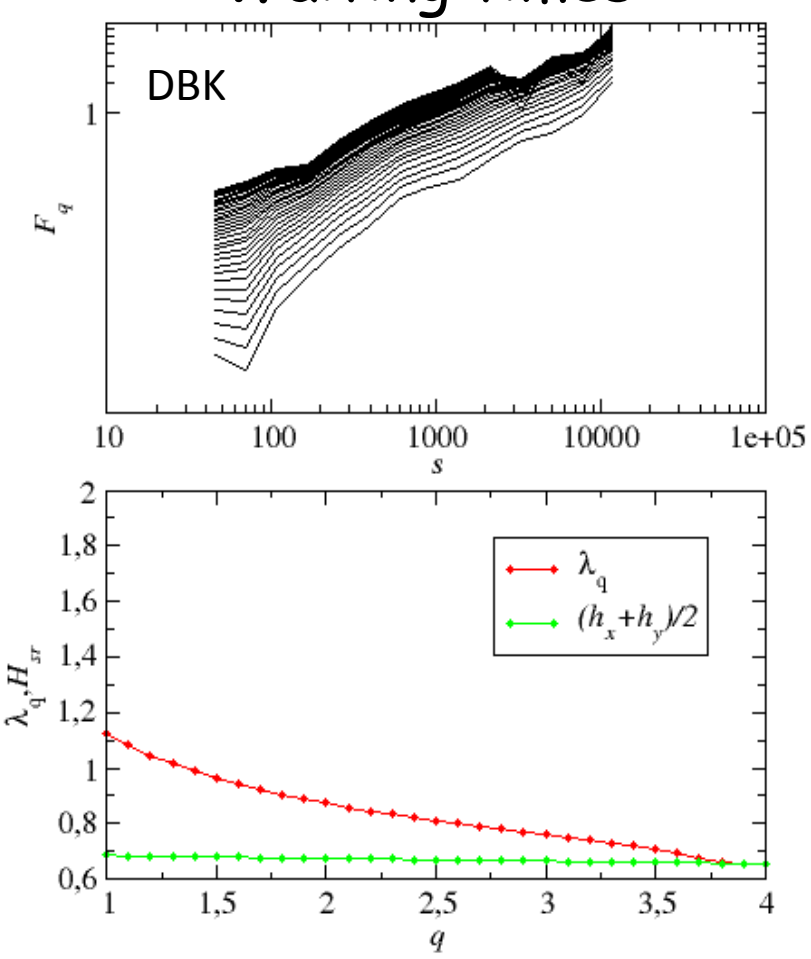


Real-world signals

Volumen returns vs. returns



Price increments vs. Waiting times



The End